

T H E S I S S T A T E M E N T

of the dissertation entitled

”‘Optimal Control Problems under Uncertainty’”

for the acquisition of PhD degree of the Sofia University

by Boyan Kolev Stefanov

in Mathematics (Operations Research)

The review is prepared by Prof. DSci Nadia Peycheva Zlateva, member of the scientific jury for the procedure, according to Order No. RD 38-129/01.03.2024 of the Rector of the Sofia University.

1. General description of the dissertation

The dissertation presented by Boyan Stefanov has a volume of 91 pages and is written in English. It starts with Resume, Declaration of Authorship, Dissemination, Acknowledgements and Introduction of 9 pages. They are followed by substantive chapters, Conclusion and Bibliography of 53 (unnumbered) titles. In addition, four author’s papers containing results of the dissertation – tho published, one accepted for publication and one submitted – are indicated at page iii.

The dissertation contains scientific results which are an original contribution to the scientific field with the possibility of being continued in future research. The reference to already known results is exhaustive and correct. The content is clear and sufficiently detailed. A unified approach to the topic is demonstrated and a good ability to combine different methods and techniques of proving the main results is shown.

2. Data and personal impressions about the candidate

Boyan Stefanov is a graduate of Faculty of Mathematics and Informatics (FMI) of Sofia University (SU). He graduated from the Bachelor programme in Mathematics in 2017 and the Master programme in Optimization in 2019. From February 2020 to February 2023, he is a full-time doctoral student in the Operations Research doctoral program of FMI(SU) with scientific supervisor Prof. DSci Mikhail Krastanov.

I have known Boyan Stefanov since he was a MSc student. I have good impressions of him as a colleague with a dedication to Mathematics. I know his work as a member of the examination boards of his specialty exams, as well as a listener of the talks he gave at various scientific forums to which he presented the new results obtained. During the second term of the academic 2023/2024 year Boyan Stefanov is a part-time assistant at FMI(SU) in Operations Research discipline to which I am a lecturer.

3. Analysis of the dissertation

Dissertation of Boyan Stefanov is in the field of Optimal Control. Certain problems of the Optimal Control over an infinite time horizon are studied within both continuous and discrete-time frameworks. The controllable systems under uncertainty (disturbances) are considered. The aim is to obtain necessary conditions as well as sufficient conditions for optimality.

In **Chapter 1** two-person non-cooperative linear-quadratic (LQ) games in continuous time on an infinite-time horizon are considered. Here it is established a sufficient condition for a saddle point, which also corresponds to the Nash equilibrium, in an infinite time horizon LQ game, under constraints on the control actions of the minimizing player. Such a condition is important for determining control strategies under uncertainty. Although the results are framed within the context of saddle points (Nash equilibria), they are equally applicable to Stackelberg game formulations, which are often more suitable for certain economic models. The proposed approach is illustrated by a simple monetary policy model, formulated in continuous time, in the published article on the subject.

In **Chapter 2**, the focus is on discrete-time LQ games over an infinite horizon, again with a special emphasis on scenarios where constraints are applied to the actions of the agent seeking minimization. Although some similarities to the continuous case, here the proofs are more difficult and require different techniques. Using Bellman's principle of optimality, sufficient optimality conditions for the linear-quadratic discrete-time game are obtained. There is some inaccuracy in the formulation of the Bellman function used in Theorem 2.1, but it can be removed in a way that does not affect the result. The practical application of the approach is illustrated through a model detailing the short-period dynamics of an F-16 aircraft.

In **Chapter 3** are derived optimality conditions of Pontryagin's maximum principle type for discrete controllable systems where the objective is to minimize the maximum adverse impact of disturbances. Additionally, specific optimal control discrete-time problems without disturbances, are considered and under appropriate assumptions a new sufficient optimality condition is proved.

In brief, the thesis presents a general framework for considering practical problems in real conditions, with special attention being paid to the inclusion in the model of uncertainty, which is common in dynamic systems.

The **bibliography** is comprehensive and shows a thorough knowledge of the field. The titles in it are arranged alphabetically by the family name of the first author but they are unnumbered. There are hyperlinks to them in the pdf file of the dissertation, but the reader of the paper version is hampered. As a technical omission, I can note that in places in the referred titles there is an inversion of the author's name and surname, which could have been avoided.

4. Approbation of the results

In **Conclusion** part of the dissertation the main contributions clearly and explicitly are stated. On page iii four publications containing results from the dissertation are listed:

[1] M. I. Krastanov, R. Rozenov, and B. K. Stefanov, On a constrained infinite-time horizon linear quadratic game, *Dynamic Games and Applications*, Volume 13, Issue 3, Birkhäuser, 2023;

[2] M. I. Krastanov, B. K. Stefanov, On Decision Making under Uncertainty, *Lecture Notes in Computer Science*, 209-220, ISSN 0302-9743, Springer, 2023;

[3] M. I. Krastanov, B. K. Stefanov, A Sufficient Condition for a Discrete-Time Optimal Control Problem, *Lecture Notes in Computer Science*, Springer, 2023;

[4] M. I. Krastanov, R. Rozenov, and B. K. Stefanov, On a Linear-Quadratic Game with Constrained Control over a Discrete Infinite-Time Horizon, 2023.

Papers [1] and [2] are published, [3] is accepted for publication, while [4] is submitted. Boyan Stefanov has personally given an impressive number of 13 talks in which he presented the results within his dissertation at scientific seminars and conferences.

It makes a good impression that, in addition to the scientific supervisor Prof. M. Krastanov, co-author of two of the articles is also Dr. R. Rozenov, who works at the International Monetary Fund. I have no reason to doubt the equal contribution of Boyan Stefanov in all above mentioned articles. The submitted dissertation and related publications contain original results and are free of any plagiarism.

5. Qualities of the Abstract

The **Abstract** is in Bulgarian, with a volume of 39 pages and 59 titles of cited literature. It comprehensively and correctly reflects the results in the dissertation. At the end of the **Introduction** (pages vii-ix) and in the **Author's Annotation** (page 22), the contributions of the dissertation work are clearly indicated. There are some linguistic inaccuracies in the translation from English, but they do not affect the overall good impression of the abstract.

The numbering of theorems, definitions, etc. in the abstract does not match their numbering in the dissertation and this makes difficult to trace the correspondences.

6. Conclusion

Boyan Stefanov's dissertation is an original study in the field of Optimal Control. The obtained results are new, interesting and have potential for further development.

Based on the above analysis, **I confirm** that the submitted dissertation work and the scientific publications related to it, as well as the quality and originality of the results presented in them, meet the requirements of the Bulgarian legal regulations for the candidate's acquisition of the PhD in Mathematics. In particular, the candidate satisfies the minimal national requirements in the professional field and no plagiarism was found in the scientific works submitted by him within the procedure.

Based on the above, **I recommend the scientific jury to award Boyan Kolev Stefanov PhD in Mathematics (Operations Research).**

May 10, 2024

/Prof. DSci Nadia Zlateva/